Let's refer back to [last week's post](). We discussed why it is important to fully understand what you are doing and why you are doing it, especially while using an innovative method. We talked about it using a combinatorics example.

Let's revisit it here:

Question 1: What is the probability that you will get a sum of 8 when you throw three dice simultaneously?

We discussed the ways of obtaining various sums. The regular way of obtaining a sum of 8 is enumerating all the possibilities. An innovative way was using 7C2 (as discussed last week).

Now the question we raised last week was this: Why does this method fail for the sum of 9?

Think about why this method works for the sum of 8 (and others before it) – when you split 8 in 3 groups, you first give 1 each of the three dice (since each dice must show at least 1) and then split the rest of the 5 in 3 groups. What happens when we try to do the same for the sum of 9? When we give 1 to each die, we are left with 6. Now when we split 6 among 3 groups, three of the cases will look like this:

6, 0, 0

0, 6, 0

0, 0, 6

What does this imply? It implies that 2 dice show 1 each and one die shows 7! Of course, no die can show 7 so these 3 cases need to be removed. So out of 8C2 = 28 cases we need to remove 3 and we get 25 cases. That is the correct answer!

Similarly, how will you adjust for the sum of 10? There will be cases where the split is like this: (7, 0, 0) or (6, 1, 0). These do not work since the maximum a die can show is 6. So you need to remove 9 cases (3 arrangements of 7, 0, 0 and 6 arrangements of 6, 1, 0) from the obtained sum of 9C2 = 36.

So there are 36 – 9 = 27 ways in which you can obtain a sum of 10.

Let's answer the original question now:

No. of cases in which the sum will be 8 = 21

Total number of cases = 6*6*6 = 216 (since the 3 dice can show any one of the 6 numbers)

Probability of obtaining a sum of 8 on a throw of three dice = 21/216 = 7/72.

Let's see if this method works for 4 dice.

Question 2: If four dice are thrown together, what is the probability that the sum on them together is either 18 or 22?

Solution: Let's try and use the same method to find the sum in case of four dice.

First of all, we see that the minimum sum will be 1*4 = 4 and the maximum sum will be 6*4 = 24. The number of ways will be symmetrical about the mid-point i.e. 14.

4, 5, 6 … 12, 13, 14, 15, 16 … 22, 23, 24

So number of ways in which you can obtain 13 is the same as the number of ways in which you can obtain 15. The number of ways in which you can obtain 12 is the same as the number of ways in which you can obtain 16 and so on.

No. of ways in which you can obtain 18 will be the same as the number of ways in which you can obtain 10. To get a sum of 10, we give 1 to each die and then split the leftover 6 among 4 groups. We can do that in 9C3 ways [As discussed last week, we obtain (6+3)!/3!*6! by using method II of question 2 discussed in this post.]

But 9C3 includes 4 cases which look like this:

6, 0, 0, 0

0, 6, 0, 0

0, 0, 6, 0

0, 0, 0, 6

This is not acceptable since no die can show more than 6. Hence, number of ways of obtaining a sum of 10 = 9C3 – 4 = 80 ways

No. of ways in which you can obtain a sum of 22 is the same as the number of ways in which you can obtain a sum of 6. To obtain a sum of 6, give 1 to each of the 4 dice and split the remaining 2 in 4 groups in 5C3 = 10 ways

No. of ways in which you can obtain 18 or 22 = 80 + 10 = 90

Total no. of cases = 6*6*6*6

Probability of obtaining a sum of 18 or 22 = 90/6*6*6*6 = 5/72

To sum it all up, it’s great to use innovative methods/shortcuts but there is a caveat – ensure that you fully understand the reason the shortcut works.